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POSSIBLE DETERMINATION OF $\bar{q}_0$ USING LUNAR OCCULTATIONS AND LASER RANGING OBSERVATIONS

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ABSTRACT

Using recent determinations of the atomic and tidal lunar acceleration, we propose a model to evaluate $\bar{q}_0$. Our conclusion is that the universe is open for values of H_0 close to 50 km s^{-1} Mpc^{-1}.
Subject headings: cosmology — Moon — occultations

I. INTRODUCTION

The search for the true value of the deceleration parameter $\bar{q}_0(\bar{q}_0 \leq \frac{1}{2}$, open universe; $\bar{q}_0 > \frac{1}{2}$, closed universe) has motivated most of the past and present research in cosmology. However, since no final answer has yet been obtained (Gott *et al.* 1974), the search continues and new methods are proposed and investigated.

Using the covariant theory of gravitation (Canuto *et al.* 1977), we present in this paper the results of a determination of $\bar{q}_0$ based on recent data on the time evolution of the period of the Moon. We conclude from our analysis that the universe is open.

II. THE DATA

The braking action of lunar tides causes the Earth to lose spin angular momentum; consequently, the Moon's orbital angular momentum increases and so does its distance from the Earth and its period of revolution, $P = 2\pi/n$.

The time variation of n (indicated by $\dot{n}$) over long periods of time has been determined using both *atomic* and *gravitational* clocks, the latter ones measuring ephemeris time. If the results provided by the two clocks were identical, we would have to conclude that the physical constants at the basis of the mechanisms on which the two clocks operate (*electrodynamics* involving e and $\hbar$ in the case of the atomic time, and *gravitation* involving the gravitational constant G in the case of ephemeris time) do not change with respect to one another as time progresses.

If, on the other hand, the measurements of the lunar period with the two different clocks yield results meaningfully different (i.e., above the uncertainties of the measurements), then we would have to conclude that gravitational quantities change with respect to atomic clocks.

Equally valid would be the conclusion that atomic quantities change with respect to ephemeris time. However, since the clocks most commonly used are the atomic ones, we shall confine our attention to the first interpretation.

The history of the measurements of $\dot{n}$ has been presented on more than one occasion, and we refer the reader to the appropriate literature (see Muller 1978, for example).

In this paper, we shall use four *atomic values* for $\dot{n}$ as from the work of Calame and Mulholland (1978*a*,*b*), Williams, Sinclair, and Yoder (1978) and Van Flandern (1980) and five *gravitational values* as from Morrison and Ward (1975), Muller (1978), Lambeck (1977), and Goad and Douglas (1978).

III. THEORETICAL EXPRESSION FOR $\Delta\dot{n}/n$

We shall now derive a theoretical expression for the difference between $\dot{n}$(atomic) and $\dot{\bar{n}}$(gravitational) in terms of the quantity $H_0 t_0$, where H_0 is the Hubble constant and t_0 is the age of the universe.

In the covariant theory of gravitation (Canuto *et al.* 1977), the *atomic* and *gravitational time* intervals dt and $d\bar{t}$ are related by the gauge function $\beta(t)$, i.e.,

$$d\bar{t} = \beta(t)dt \tag{3.1}$$

with the constraint

$$\beta GM = \text{constant}\,, \tag{3.2}$$

where G is the gravitational constant and M the macroscopic mass of the object under consideration.

Because of (3.1) we also have

$$n = \bar{n}\beta\,, \tag{3.3}$$

so that when evaluated today, $\beta_0 = 1$, we obtain

$$\frac{\Delta\dot{n}}{n} \equiv \frac{\dot{n} - \dot{\bar{n}}}{n} = \frac{\dot{\beta}}{\beta}\,. \tag{3.4}$$

As in all previous papers, the gauge function $\beta(t)$ will be parametrized as

$$\beta(t) = (t/t_0)^{-\epsilon} \tag{3.5}$$

so that finally

$$\frac{\Delta\dot{n}}{n} = -\frac{\epsilon}{t_0} = -\frac{5.1h}{H_0 t_0}\frac{\epsilon}{10^{11}\ \text{yr}}\,. \tag{3.6}$$

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Next, we shall express the quantity $H_0 t_0$ in terms of the deceleration parameter $\bar{q}_0$, related to the spatial curvature by

$$k = \bar{R}_0{}^2 \bar{H}_0{}^2 (2\bar{q}_0 - 1) . \tag{3.7}$$

Because of (3.1), the scale factors of the universe in atomic and gravitational units are related by

$$\bar{R} = \beta R , \tag{3.8}$$

so that today

$$\bar{H}_0 = \dot{\beta}_0 + H_0 \tag{3.9}$$

where $\bar{H}_0$ and H_0 are the Hubble constants in the two units. From (3.1) and (3.5), it also follows that

$$\bar{t} = \frac{t_0}{1-\epsilon}\left(\frac{t}{t_0}\right)^{1-\epsilon} , \quad \bar{t}_0 = \frac{t_0}{1-\epsilon} . \tag{3.10}$$

Multiplying (3.9) by $\bar{t}_0$, we finally obtain

$$\bar{H}_0 \bar{t}_0 = (1-\epsilon) H_0 t_0 + \epsilon . \tag{3.11}$$

The quantity $\bar{H}_0 \bar{t}_0$ has been tabulated versus $\bar{q}_0$ by Sandage (1961); $H_0 t_0$ is therefore a known function of $\bar{q}_0$, and so is the theoretical value $\Delta\dot{n}/n$, equation (3.6).

Using the observational values cited before and relation (3.3), we have performed a least squares fit analysis following the method first outlined by Muller (1978; his Fig. 16). The result is

$$\left(\frac{\Delta\dot{n}}{n}\right)_{\text{obs}} = (3.1 \pm 1.0)\frac{10^{-11}}{\text{yr}} . \tag{3.12}$$

Inserting (3.6) and (3.12) in Figure 1, we reach the following conclusions.

1. $\epsilon = -1$. In this case, the data clearly favor an open universe. In fact, a value $\bar{q}_0 \geq \frac{1}{2}$ would be allowed only if $H_0 < 5$, totally outside the range of currently accepted values. Within the uncertainties of the Hubble constant, we conclude that the value $\epsilon = -1$ is allowed only if the universe is open.

2. $\epsilon = -\frac{1}{2}$. This value was proposed by Canuto and Hsieh (1978) on considerations based on the blackbody radiation. In this case, we cannot exclude a closed universe, but an open universe is still favored, at least for values of $H_0 \gtrsim 50$. We therefore conclude that in both cases an open universe is favored over a closed one. It is important to stress that this conclusion is the same as the one arrived at from the study of the cosmological tests, as discussed in Canuto and Hsieh (1979), Canuto, Hsieh, and Owen (1979), and Canuto and Owen (1979).

IV. THE GRAVITATIONAL CONSTANT G

In discussing a possible difference between $\dot{n}$(atomic) and $\dot{n}$(gravitational), it has become customary to conclude that a non-null result *necessarily* implies a time-varying gravitational constant G. This is not necessarily the case, however.

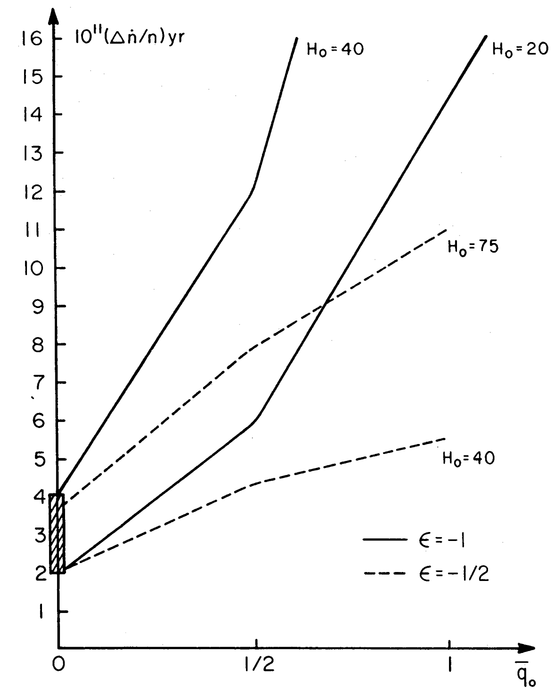

FIG. 1.—Theoretical values of $\Delta\dot{n}/n$ versus the deceleration parameter $\bar{q}_0$, for different values of H_0 and for the two gauges $\epsilon = -1, -\frac{1}{2}$. Also plotted is the observational value from equation (3.12).

In fact, a result like (3.12) can equally well be explained even with a constant G. From (3.2), (3.4), and (3.12), *all* we are allowed to conclude is that

$$\frac{(GM)^{\cdot}}{(GM)} < 0 . \tag{4.1}$$

Since a variation of $M = mN$ is in principle as possible as a variation of G, we cannot proceed any further after establishing inequality (4.1), *unless* we introduce information regarding either G or M, in addition to that derivable from the lunar observations themselves.

V. DISCUSSION AND CONCLUSIONS

The gist of the present paper is to show that *given a reliable difference* between $\dot{n}$(atomic) and $\dot{n}$(gravitational), a theoretical framework exists capable of extracting information of cosmological significance. The deceleration parameter $\bar{q}_0$, the most sought after parameter in cosmology, is the quantity we have focused on.

Considering that other methods for the determination of $\bar{q}_0$ have been vitiated by evolutionary effects difficult to quantify, it seems that the present method can offer an interesting alternative since the remaining errors in the observational data will be brought under control and reduced in size as more data become available with time. (See, for example, Table 2 of Williams, Sinclair, and Yoder 1978.)

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